

# Modelling the SN-curve for PM steels using a fracture mechanics approach

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## Abstract

Whenever designing a component it is important to properly take fatigue related phenomena into account, both to ensure safe operation, but also to allow PM to be used in ever more demanding applications. In previous papers it was shown how the endurance limit of PM steels can be modelled using a fracture mechanics approach combined with an extreme value statistics treatment of the porosity. With this approach it was possible to include both the influence of density and stress concentrations in one comprehensive model. In this paper it is investigated how the model can be extended to include also the short life portion of the SN-curve. This is accomplished by also including fatigue crack propagation in the fracture mechanics model. The use of the model is demonstrated on a chromium based PM steel for sinter hardening applications.

## Introduction

Utilization of the full potential of a material means that the behavior of the material must be well understood. Using for instance an overly conservative design will be bad both from a cost and performance point of view, but also from an environmental perspective since resources are not optimally used.

For mechanically loaded components the fatigue strength is often the limiting factor. Therefore it is of key importance to have reliable models to describe the fatigue strength of materials and components. Not least to be able to use PM materials for ever more demanding applications.

In many cases the fatigue strength is directly related to defects, such as inclusions in the material [1]. For PM the corresponding weak points are typically the pores, which act as local stress concentrations and initiation point for fatigue cracks. Therefore it is natural to develop fatigue models for PM steels that link the porosity to the fatigue strengths of the material. This is often done in an indirect way by linking the strength to the density of the material. The problem with such an approach is that it does not provide any insight into the mechanisms of the material behavior. Previous investigations [2]-[5] have shown that both the influence of density and the notch effect in fatigue of PM steels can be modelled by combining extreme value statistical analysis of the porosity with fracture mechanics.

Those investigations focused on the long life, after the knee point of the SN curve. However, in many cases it is also of importance to have a description of the fatigue behavior before the knee, to model for instance load spectra. The subject of this paper is therefore to extend the fracture mechanics approach to also account for fatigue crack growth in order to describe the full SN curve.

## Model

The basic assumption of the model is that fatigue cracks initiate at the largest pore in the highly stressed volume of a PM steel, and that fracture mechanics can be used to analyse the influence of porosity. The porosity is described using extreme value statistics to obtain the largest pore in a given volume. This porosity distribution will then be density dependent, different densities yield different size pores, and therefore the model includes the density dependence of PM in a natural way. Also, in the porosity distribution, the highly stressed volume,  $V_{90}$ , will appear, which means that the notch effect will also be accounted for. In a previous publication [5] it was demonstrated how this framework could be used to include both the effect of density and stress concentrations as a comprehensive model for the endurance limit. Now by looking at the growth rate of fatigue cracks, a similar model will be developed to model the short life portion of the SN curve as well.

The growth rate per cycle of fatigue cracks,  $da/dN$ , can typically be described as a function of the applied stress intensity factor range,  $\Delta K$ , where the stress intensity factor takes into account both the applied load and the crack length.  $\Delta K$  can for simple geometries be calculated as:

$$\Delta K = \sigma_a \sqrt{\pi a} \cdot Y(a) \quad (1)$$

where  $a$  is crack length, and the function  $Y(a)$  compensates for the influence of geometry. In this case the solution for a corner crack in a rectangular cross section subjected to bending was used corresponding to the experimental setup. Formulas are given in [6]. Since the investigated case is for load ratio  $R = -1$ , the stress range is taken as the amplitude:  $\Delta\sigma = \sigma_a$  for the positive part of the stress cycle.

A typical empirical relation between  $\Delta K$  and  $da/dN$  is shown in Figure 1, where for low  $\Delta K$  values, a threshold is typically found, and for higher values a linear relation is found when plotted using log-log scales. The latter portion of the curve is often described using Paris' law stated as:

$$\frac{da}{dN} = C \cdot \Delta K^m \quad (2)$$

The fatigue life of a component can then be calculated by integrating the crack growth rate from initial defect size,  $a_0$ , to its final size,  $a_f$ , see for instance [6]. Denoting the number of cycles to failure with  $N_f$  the following relation is obtained:

$$\sigma_a^m N_f = \frac{1}{C} \int_{a_0(\rho)}^{a_f} \frac{da}{[\sqrt{\pi a} \cdot Y(a)]^m} = \frac{f(\rho, m)}{C} \quad (3)$$

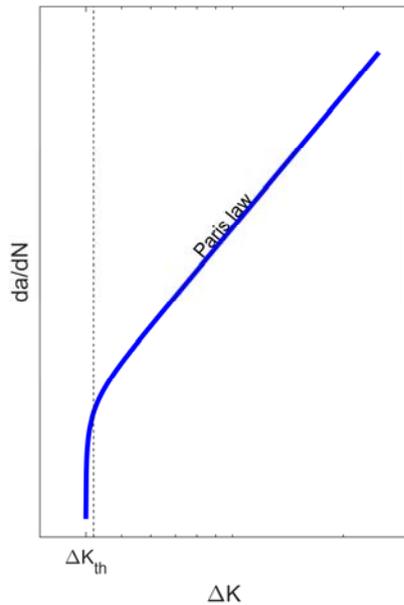
Here it was assumed that the initial defect size is density dependent, corresponding to different sized pores. It is obvious that the above equation is closely related to the Basquin relation:

$$\sigma_a^m N_f = C_w(\rho) \quad (4)$$

often used to describe the finite life part of the SN curve of a material. Comparing the equations give:

$$C_w(\rho) = \frac{f(\rho, m)}{C} \quad (5)$$

where the different meaning of  $C_w$  and  $C$  must be observed. It should also be noted that the model predicts that the Wöhler exponent,  $m$ , does not depend on density.



**Figure 1. Schematic illustration of crack growth rate as a function of stress intensity factor, logarithmic scales.**

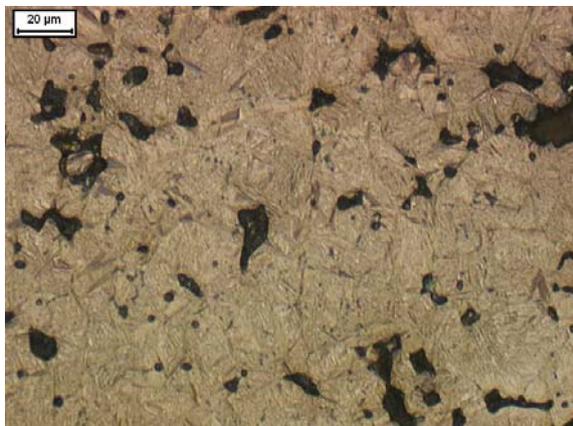
Previous papers [2]-[5] have described how the porosity can be modelled using extreme value statistics, and more specifically the Gumbel distribution. By doing a metallographic analysis of the pore structure, the largest pore in a given volume can be described using the following expression:

$$A_{\alpha} = \lambda + \delta \left[ \ln \frac{V_{90}}{V_0} - \ln(-\ln \alpha) \right] \quad (6)$$

where  $\lambda$ ,  $\delta$  and  $V_0$  are material and density related parameters.  $V_{90}$  denotes the volume of a part with at least 90% of the peak stress, also referred to as the highly stressed volume. The final parameter  $\alpha$  is a probability value, taken as  $\alpha=0.5$  for the median.

### Experimental investigation

For the experimental investigation ISO 3928 test bars were manufactured from a premix consisting of Astaloy CrM (Fe-3Cr-0.5%Mo pre-alloyed)+0.45%C-UF4+0.8%Amide wax. Three different green densities were used: 6.90, 7.00 and 7.15 g/cm<sup>3</sup> respectively. The test bars were sintered at 1120°C for 30 min in a 90/10 N<sub>2</sub>/H<sub>2</sub> atmosphere with addition of 0.2%CH<sub>4</sub>. Sinter hardening was applied with a cooling rate of around 2.5 K/s, resulting in a fully martensitic structure, see Figure 2.



**Figure 2. Fully martensitic structure of Astaloy CrM+0.45C.**

In a previous investigation [2] the largest pores for the different densities, and the standard test bar geometry, were determined. The values are given in Table 1. As can be seen in the table there is a clear dependence on density, where increasing the density yields smaller pores.

**Table 1. Initial crack length for different densities.**

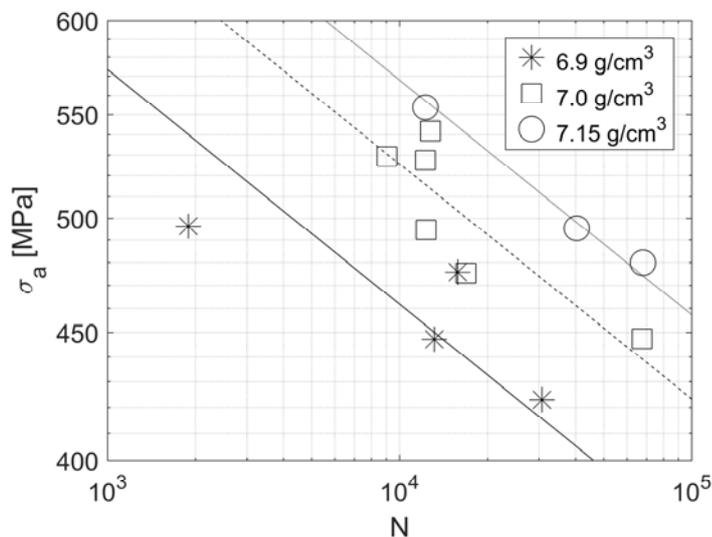
$\rho$ [g/cm <sup>3</sup> ]	$a_0$ [ $\mu$ m]
6.90	59.1
7.00	51.9
7.15	42.9

Fatigue testing was done in plane bending, with a load ratio of  $R = -1$  and a load frequency of around 30 Hz. Failure was defined as an increase in compliance by 2.5%, corresponding to a final fatigue crack length of around 1.8 mm. Figure 3 shows an example of the final crack, with the initiation point at the upper left corner.



**Figure 3. Cross section of a test bar with the final fatigue crack marked.**

Fatigue testing was done with different stress amplitudes, resulting in SN-curves as shown in Figure 4. In the figure the estimated Wöhler lines according to (4) are also included, where it was assumed that the slope is independent of density.

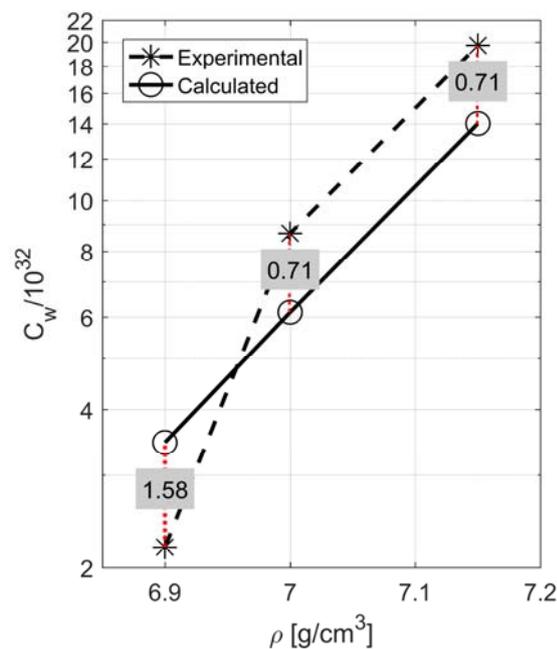


**Figure 4. Experimental SN-curves of sinter hardened Astaloy CrM+0.45C.**

From the experimental SN-curves the Wöhler exponent was calculated as  $m=10.6$ , which was then used in eq. (3) to numerically calculate  $f(\rho,m)$ . By combining the experimental values of  $C_w$  with  $f(\rho,m)$  a value of the crack growth parameter  $C$  could finally be calculated by taking the average of the measurements for the different densities. The calculated value was  $C=1.05 \cdot 10^{-29}$ .

Next, by taking the estimated  $C$ -value together with  $f(\rho,m)$  the calculated  $C_w$  values were obtained. These values, compared to the experimental ones directly from Figure 4 are shown in Figure 5. Since the parameter  $m$  is kept constant, differences between calculated and measured values for  $C_w$  directly reflect the difference between the fatigue life for a given stress obtained directly from the experimental SN curve to that from the calculated curve according to the presented model.

From Figure 5 it is seen that the difference is a factor between 0.7 and 1.6, thus using the model would give a fatigue life between 30% shorter and 60% longer than the direct experimental values. It can also be observed that it is mainly the point from the lowest density that gives the largest difference between model and experiments.



**Figure 5. Calculated C values compared to experimental ones.**

### Discussion

The results show that the model is in reasonable agreement with the experiments, and is able to capture the general influence of the density on the fatigue strength. The difference between the measured and the calculated cycles to failure is between 0.71 and 1.58, which corresponds to a difference in the stress direction of roughly 3-5%. This is a similar difference as previously found when modelling the endurance limit, see for instance [2]-[5].

Looking at Figure 5 it also seems that the main difference between model and experiment occurs at the lowest density ( $\rho=6.9$  g/cm<sup>3</sup>). Given the limited number of data points available, 4 points for  $\rho=6.9$  g/cm<sup>3</sup>, as well as the natural scatter in the SN-curves, there is also some uncertainty in the experimental curves. Further testing should therefore be done in order to limit the experimental error. Future studies should also include notched test bars to see if the notch effect can be taken into account. Previous studies such as [3] and [5] have shown that for the endurance limit this effect is covered by the influence of the highly stressed volume on the maximum pore size, and it is interesting to see if this is also the case for the SN-curves.

Though the initial results are promising, there are some issues that have to be looked more deeply into. For instance, in [7] it was shown that the fatigue crack propagation in PM steels is largely influenced by coalescence of micro cracks ahead of the main crack. This effect is not taken into account in the model presented here, where crack growth is assumed to take place through one main crack. Especially for the initial part of the process, when the main crack is more or less of the same size as the porosity ahead of the crack front, it can be assumed that void coalescence plays an important role. Possibly it can be argued that these effects of sudden jumps in the crack front on average cancel out and that the effect is implicitly included in the values of the parameters in equation (2). But this requires further investigations, and possibly an extension of the model.

Also the model prediction that the Wöhler exponent is independent of density should be further explored by more experimental data.

## Conclusions

It was found that it is possible to link the porosity of a PM steel to the behavior of the SN curves by modelling crack growth using a fracture mechanics approach. The model was shown to capture the fundamental behavior of the SN curves for PM steels, including the effect of density, with reasonable accuracy. More testing is necessary to investigate the limits of the model and see for instance what the effect of pore and micro crack coalescence is. It would also be of interest to see how well the notch effect can be taken into account.

## References

- [1] Murakami Y., *Metal Fatigue: Effects of Small Defects and Nonmetallic Inclusions*, Elsevier, 2002
- [2] Andersson M. and Larsson M., *Linking Pore Size and Structure to the Fatigue Performance of Sintered Steels*, Proceedings of PM2010 World Congress, Florence, vol. 3, 2010
- [3] Andersson M., *The role of porosity in fatigue of PM materials*, Powder Metallurgy Progress, vol. 11, pp. 21-31, 2011
- [4] Andersson M., *The influence of notches on fatigue of heat treated sintered steel*, Proceedings of Euro PM2013, Göteborg, vol. 3, p. 373-378, 2013
- [5] Andersson M., *Fatigue of Heat Treated PM Steels using a Fracture Mechanics Approach*, Proceedings of World PM2016, Hamburg, Session 27: Fatigue I, 2016
- [6] Anderson T.L., *Fracture Mechanics, Fundamentals and Applications*, 2nd edition, CRC Press, 1995
- [7] Kabatova M. et.al., *Microcrack nucleation, growth, coalescence and propagation in the fatigue failure of a powder metallurgy steel*, Fatigue and Fracture of Engineering Materials & Structures, vol. 32, pp. 214-222, 2009