ZaKo3D – Simulation Possibilities for PM Gears

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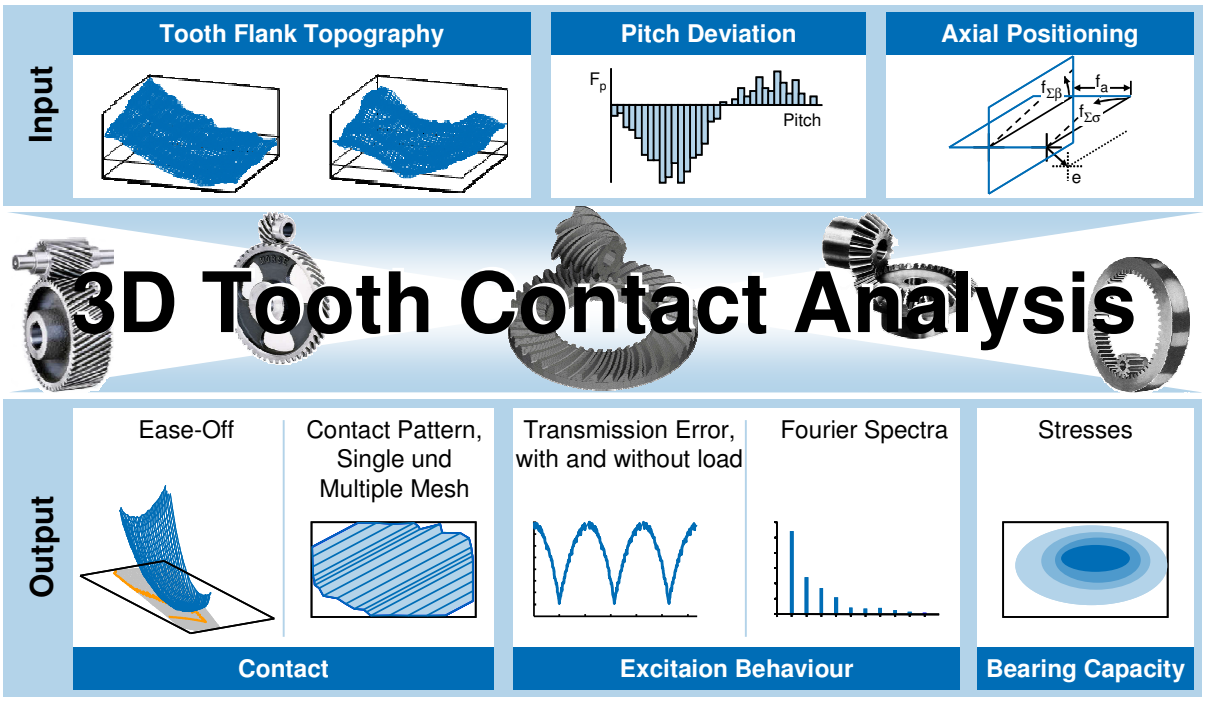
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1 Abstract

Powder metal technology is an alternative manufacturing method for the series-production of gears. But the properties of PM and regular steel gears differ. Especially the values for Young's Modulus, Poisson's Ratio and the hardness are smaller, the surface is densified and the tolerance zone is wider. Since PM gears are not state of the art yet, the properties for these gears are not taken into account by regular tooth contact analysis software.
In order to design gears for the manufacturing with PM technology which will show good noise and bearing behaviour, the design has to be tested by tooth contact analysis software that takes the PM characteristics into account. This paper shows the program ZaKo3D of WZL which is able to take the characteristics into account using a FE-based approach. Using ZaKo3D a case study for the design of the micro geometry of an automotive PM gear was carried out. This study, including the PM gear modelling, the sensitivity analysis and the derivation of flank modifications is presented in the paper.

2 Description of ZaKo3D

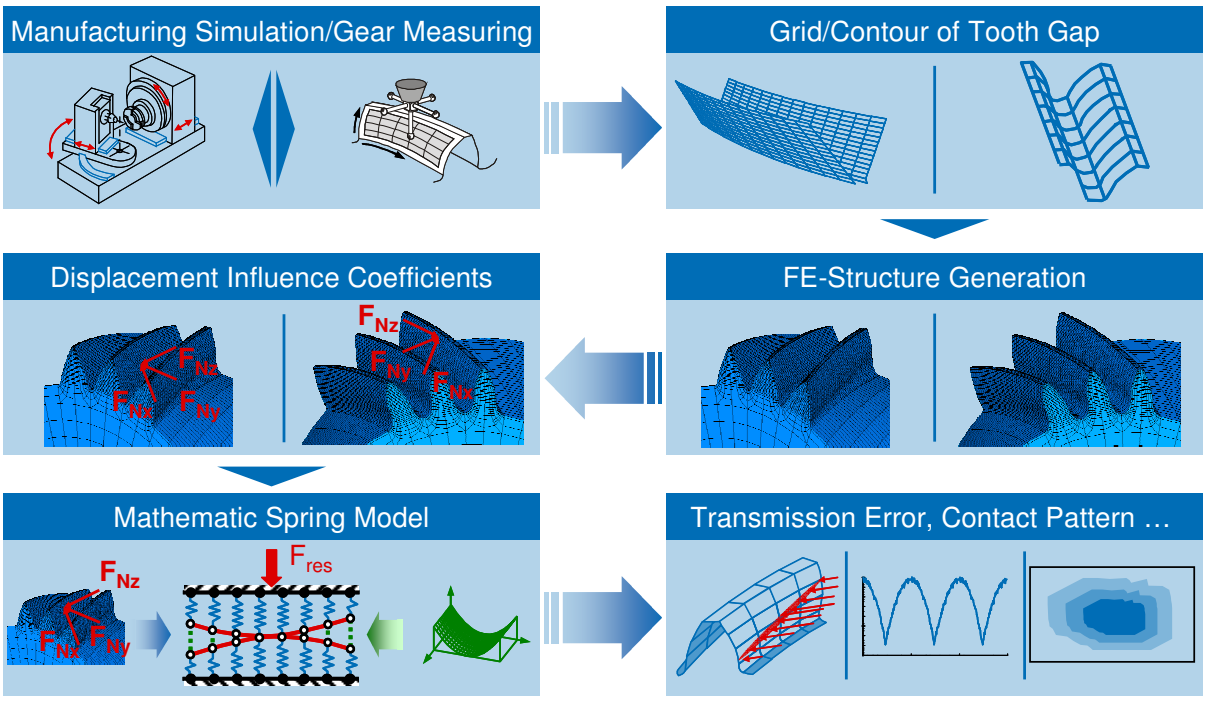
The general approach of ZaKo3D is the software based simulation of the tooth contact. Input data are geometric data of the flank and a FE-model of a gear section. Furthermore the input data at the top of Figure 1 (pitch and assembly deviations) can be considered. During the simulation the contact distances, loads and deflections on the tooth are calculated. The results of the calculation can be displayed in established diagrams to support the gear designer during the developing process.



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Figure 1: Features of the 3D Tooth Contact Analysis ZaKo3D

The operation sequence of ZaKo3D is displayed in Figure 2. At the beginning the flank geometry has to be provided. This can for example be done by a manufacturing simulation or by measurement. The flank is defined by points in Cartesian coordinates and the direction of the normal vectors in each flank point.
In order to be able to simulate under load, FE-data has to be generated out of the flank data. The FE-model contains the information about the stiffness of the gear. It is created by an automatic FE-mesh generator for gear teeth [1]. Additional to the flank geometry information about the distribution of FE-nodes inside the model and the material properties are needed. Each point of the modelled flanks is loaded with unit forces in each, x-, y- and z-direction. Using this model a standard FE-solver is used to calculate so called influence coefficients. The influence coefficients hold the information about the deflection of all points during the application of each unit force. This contains the displacement influence coefficients α_{ii} which are on the diagonal of the influence coefficient matrix as well as the cross influence coefficients α_{ij}. To complete the input data, information about the positioning of the gears is needed. ZaKo 3D supports the positioning of different gear types, containing e.g. spur gears, bevel gears and beveloid gears.



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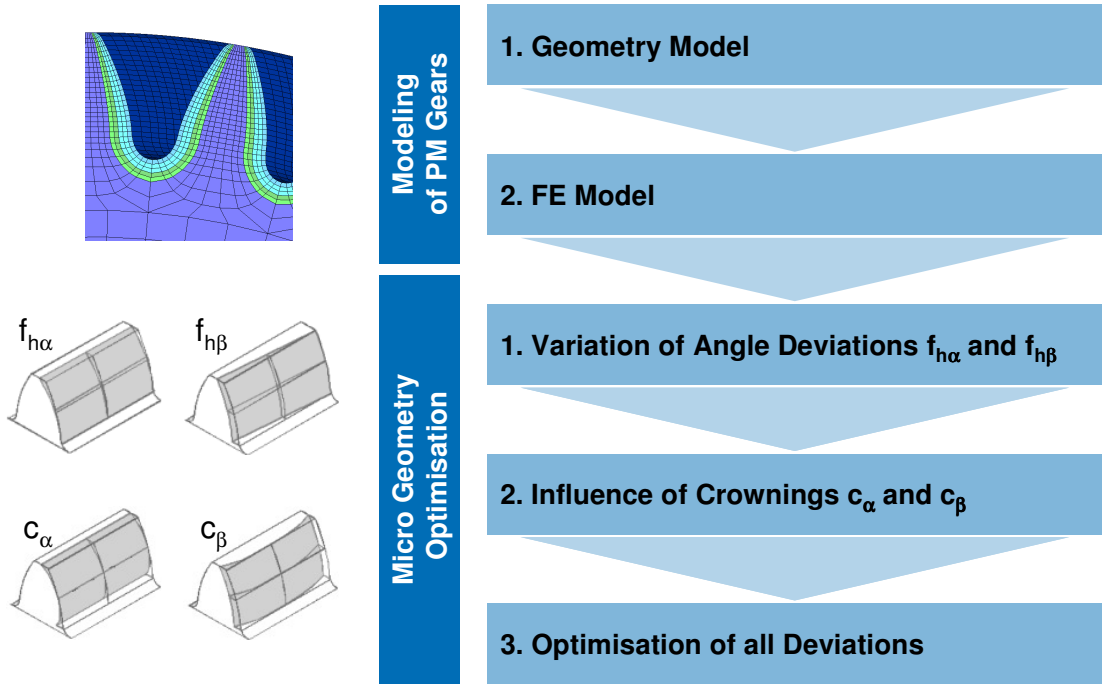
Figure 2: Operation Sequence for the Loaded 3D-Tooth Contact Analysis

The input of pitch deviations, micro geometry deviations and corrections, assembly deviations or different loads can be done by the user and is optional.
After reading in the input data the tooth contact analysis starts with the calculation of the contact distances of the flanks during the mesh. This is done for the given number of rolling positions and for each flank point of all the flanks that are in contact. With these contact distances and the information about the stiffness from the influence coefficients a mathematic spring model is defined [2]. Since the number of contact points in a rolling position and the force at each contact point influence each other it is necessary to solve the spring model iteratively.
Out of this calculations the contact pattern and the transmission error can be derived load free and under load. The transmission error of a gear is caused by geometric errors of the flanks (load free content) and deflections (load content) of the gear, and gives a good impression of the dynamic gear excitation [3]. The course of the transmission error can be displayed over time and, by performing a Fast-Fourier-Transformation, in the frequency domain.
Using the forces on the nodes and the flank area this force is applied to, the resulting pressure can be calculated. The flank area corresponding to a node is defined by the grid size [2]. The surface stress distribution on the flank has a big influence on the wear resistance of

the flanks and a reduction can lower the risk of pitting appearance and improve the flank bearing capacity. [4] Furthermore the Ease-Off, which represents the contact distances in the mesh area, is calculated load free. This output data gives information about the gear behaviour and can be used to predict the quality of the calculated gear design. This is necessary to reduce the needed number of tests.

3 Micro Geometry Optimization Method for PM Gears

Using ZaKo3D an analysis of the tooth mesh of powder metallurgical gears and the optimization of their micro geometry can be done. The procedure is presented in Figure 3.
In a first step a model of the PM gear is created. This model is divided in a geometric and a FE-model. Whereas the geometric model of a PM gear doesn't differ from a geometric wrought steel gear model, it is necessary to match the special material properties of powder metal in the FE-model.



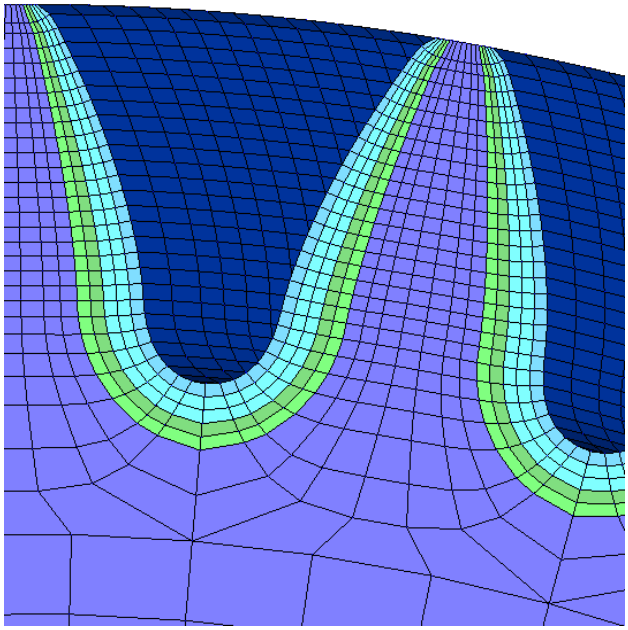
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Figure 3: PM-Gear Optimisation Using ZaKo3D

In the second step the tooth contact analysis is performed for the gear while modifying the micro geometry to take geometrical deviations and corrections into account. The micro geometry variation parameters are defined in Figure 3, lower left side. The upper two sketches illustrate the angle deviations in profile and width direction and the lower two sketches illustrate crownings in profile and width direction.
In the beginning the profile angle deviation $f_{h\alpha}$ and the width angle deviation $f_{h\beta}$ are varied in a wide range to observe their influence on bearing capacity and excitation behaviour. During this simulation step the cross influences between $f_{h\alpha}$ and $f_{h\beta}$ are not considered. The influence of manufacturing deviations in a certain tolerance class according to DIN 3962 is taken into account [5].
In the next step the cross influences of crownings c_α and c_β on the before calculated results have to be observed to find a combination of corrections which determines good noise and bearing behaviour inside a chosen tolerance class. Finally, regarding the results of the previous calculation step, a combination of flank corrections is chosen to perform a tooth contact analysis which includes all cross influences. Now the optimum combination of the parameters can be found by the iteration of the modification in a smaller range, including all cross influences.
The results in chapter 5 of this paper concern just profile angle deviations and no width angle deviations. The final modifications including the width angle deviations are presented in chapter 6.

4 Modelling of PM Gears

The presented method is applied on the fourth gear of an automotive gear box. The FE-model is created like described in chapter 2 but it has to be modified to match the properties of a PM gear. A PM gear tooth contains sections with different densities. Since the material properties of PM are dependent on the material properties of the base material and the density it is necessary to use different material properties for the densified sections [7]. Because of this different materials for these sections have to be defined inside the model. The result of this step is presented in Figure 4. The core density is 7.2 g/cm³ and the surface densification is modelled in five different layers using rising densities from the core to the outline.



	ρ [g/cm³]	E [N/mm²]	ν [-]
1st Layer	7,69	204868	0,2885
2nd Layer	7,66	201686	0,2876
3rd Layer	7,52	190171	0,2840
4th Layer	7,41	180901	0,2810
5th Layer	7,32	172906	0,2783
Core	7,2	163502	0,2749

$$E = E_0 \cdot \left(\frac{\rho}{\rho_0}\right)^{3,4} \qquad \nu = (\nu_0+1) \cdot \left(\frac{\rho}{\rho_0}\right)^{0,16} - 1$$

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Figure 4: FE-Model of the PM Gear

The material properties for the sections are chosen like presented in the right side of Figure 4. The values for the density are taken from measurements of surface densified PM preformed at the WZL. The formulas that describe the relation between Young's modulus or Poisson's ratio and the density are presented in the lower right part of Figure 4 [8].
The gear data is shown in Figure 5, left side. The normal module of the gear is $m_n = 1.7$ mm and the helix angle is $\beta = 30°15'$. The pinion has $z_1 = 35$ teeth and the gear has $z_2 = 33$ teeth. The geometric gear model for the tooth contact analysis is simulated by the generation movement of a basic rack. The input data for this calculation step contains the gear and cutting tool parameters for the corresponding manufacturing process [6]. This geometry is used for the before described FE-Model.

5 PM Gear Micro Geometry Optimisation

After creating the geometry and the FE-model of the gear the tooth contact analysis can be performed. The target parameters for this micro geometry variation are the surface stresses and the transmission error in order to optimise the bearing and noise behaviour.
The results of the stress calculation for different profile angle on pinion and gear are presented in the sensitivity analysis diagram on the right side of Figure 5. The abscissa of the diagram indicates the profile angle deviation on the pinion and the ordinate indicates the profile angle deviation on the gear. For each combination a tooth contact analysis was performed and the maximum of the resulting stress during the mesh is marked in the diagram. The height of these stresses can be identified by comparing the colour in the diagram with the colour bar next to it.

Now it is possible to identify the combination of the profile angle deviations with the minimum stress in the diagram. But in most cases this is not the best layout for a gear, since all manufacturing processes work in a certain tolerance range. In this picture these tolerance ranges are marked with a dashed box. The size of the tolerance ranges in Figure 5 represents a DIN 7 quality class. If at gear and pinion a target value of $f_{h\alpha}$ = 30 µm is chosen for the profile angle correction, which would results in the lowest value for the maximum stress σ_{min} = 1234.7 N/mm², it is possible that due to the given tolerances the manufactured gears have different profile angles and have to resist higher stresses. For the worst case inside the tolerance range ($f_{h\alpha,pinion}$ = 15 µm and $f_{h\alpha,gear}$ = 45 µm) the stress value could be up to σ_{max} = 2461.1 N/mm².

- Gear Data:

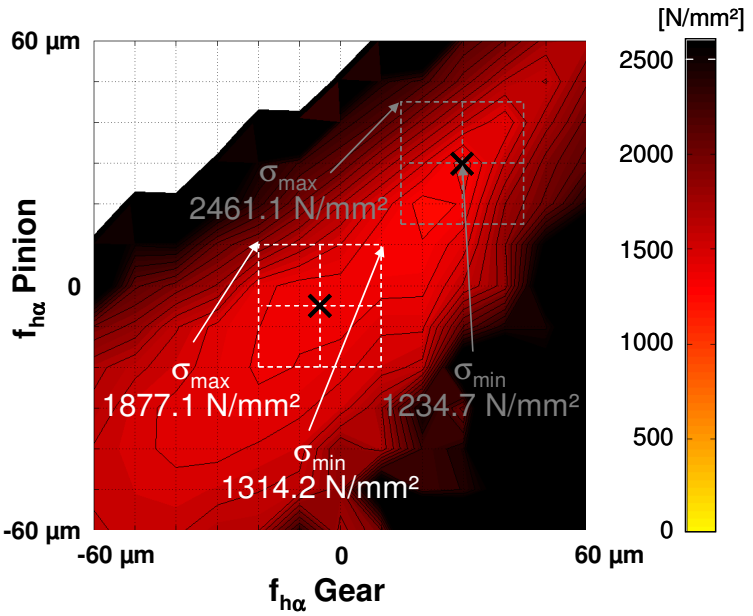
m	= 1.7 mm
z_1	= 35
z_2	= 33
b_1	= 12.7 mm
b_2	= 12.1 mm
β	= 30° 15‘ 0“
a	= 68 mm
α	= 16°

- Modifications:

c_α	= 0 µm
c_β	= 0 µm
$f_{h\beta1/2}$	= 0 µm

- Simulation:

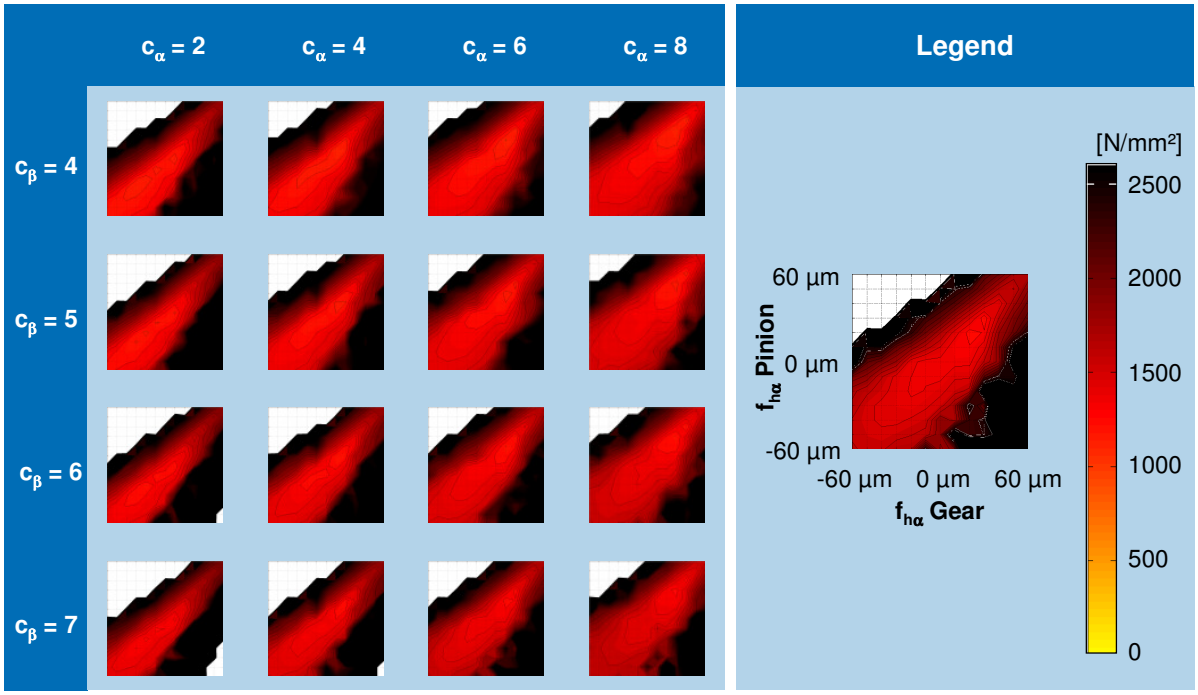
T_2	= 86.9 Nm



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Figure 5: Sensitivity Analysis Diagram

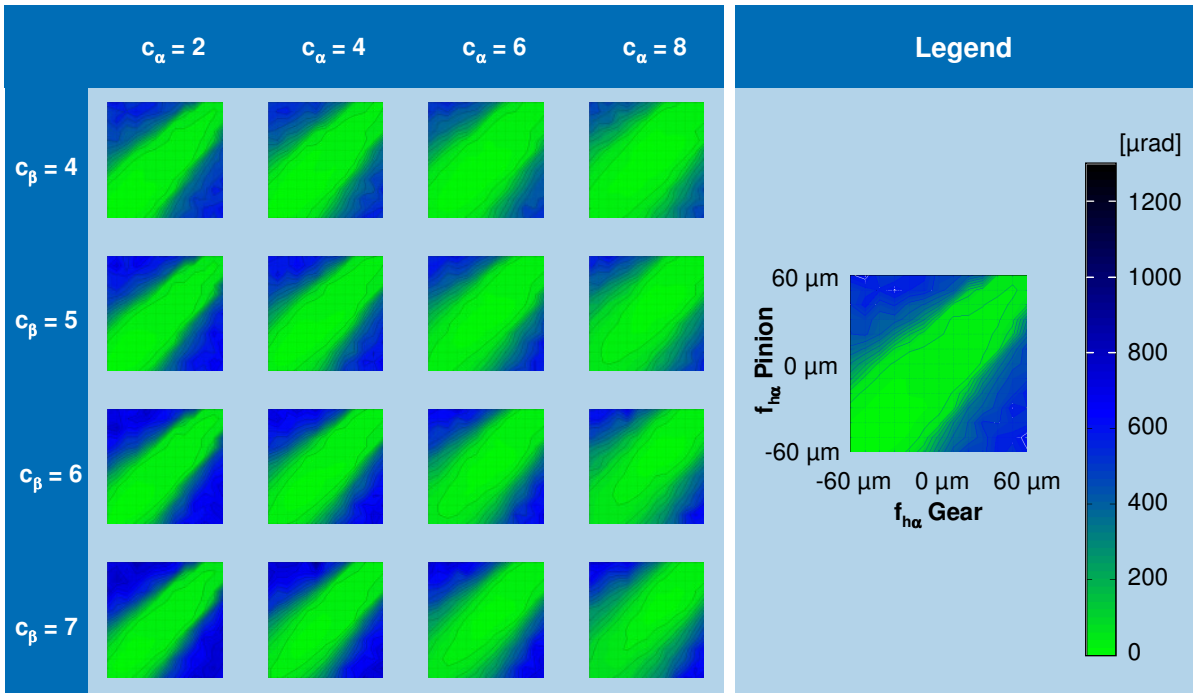
It is possible to find target values for the profile angle deviations for which the worst case in the tolerance field is lower. If the profile angle corrections of $f_{h\alpha,pinion,gear}$ = -5 µm are chosen the worst case leads to maximum stress of σ_{max} = 1877.1 N/mm². This drop of 23.7 % goes along with a rise of 6,4 % of the maximum stress for the best case to σ_{min} = 1314.2 N/mm². Since this rise is small compared to the drop of the worst case maximum stress this could be acceptable.
The same procedure is done for the width angle deviation and furthermore in profile and width direction for the transmission error, too. In the next step the angle deviations are superimposed by crownings in profile and width direction. The reaction of the sensitivity analysis diagram for profile angle deviations on the different crownings is presented in Figure 6. The indicated crownings were added on the pinion and on the gear.
For a rising value of the profile crowning the sensitivity of the maximum stress against profile angle deviations decreased. That is why bigger deviations do not necessarily lead to a big rise of the surface stresses. Furthermore the best case in the single pictures moves from the lower left corner to the upper right corner. So it moves from negative values for $f_{h\alpha}$ on both gears to positive values.



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Figure 6: Sensitivity Analysis of the Resulting Maximum Stress

A rising value for the crowning in width direction determines a rising sensitivity of the maximum stress against the profile angle deviation. An influence of the width crowning on the position of the best case in the sensitivity analysis diagram can not be observed.



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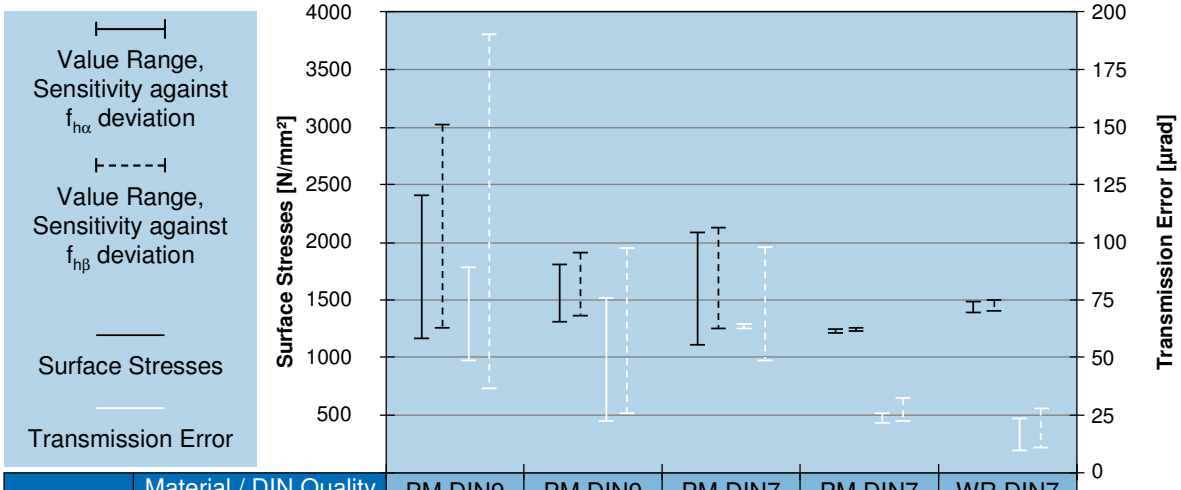
Figure 7: Sensitivity Analysis of the Transmission Error

Now it is possible to find a combination of flank corrections for which the stresses inside the tolerance field stay in a low range. The optimum micro geometry corrections for the presented gear would be c_α = 2 µm, c_β = 4 µm, $f_{h\alpha,1}$ = -30 µm and $f_{h\alpha,2}$ = -40 µm if just the influence of $f_{h\alpha}$ deviations on the surface stresses would be considered and the tolerance range would be according to DIN 7 [5]. Since the optimisation has to be done for $f_{h\beta}$ deviations and

for the consequences on other targets like the transmission error, it is necessary to make trade offs between the different target values. That's why the chosen corrections are slightly different.
To clarify this issue the influence of crownings c_α and c_β and profile angle deviations $f_{h\alpha}$ on the transmission error are presented in Figure 7. Like it was observed for the surface stresses, a rising profile crowning leads to a lower sensitivity of the maximum transmission error against profile angle deviations. This sensitivity rises with a rising width crowning. In contrast to the influence of crownings on surface stresses the position of the best value moves with both, width and profile crowning. In both cases the best value moves into the direction of positive profile angle deviations for rising crownings.
Furthermore the tolerance range is of interest. That's why it is necessary to find a compromise for these values which can be done by an iterative method. In this case the optimum corrections would be $c_\alpha = 2$ µm, $c_\beta = 5$ µm, $f_{h\alpha} = -37$ µm for both pinion and gear if a DIN 7 tolerance range is chosen. The width angle deviations differ from pinion to gear and should be $f_{h\beta,pinion} = -37$ µm and $f_{h\beta,gear} = 35$ µm. For this correction the maximum stresses are between $\sigma_{min} = 1211.8$ N/mm² and $\sigma_{max} = 1259.9$ N/mm² and the transmission error between $TE_{min} = 21.3$ µm and $TE_{max} = 32.6$ µm.

6 Results

Figure 8 displays the results for different gear designs in different quality classes. In the diagram are the value ranges for surface stresses (black) and transmission error (white) depending on the deviations in profile direction (straight line) and width direction (dashed line).



	Material / DIN Quality	PM DIN9	PM DIN9	PM DIN7	PM DIN7	WR DIN7
Optimized Geometry	$f_{h\alpha,1}$ / $f_{h\alpha,2}$	0 / 0	-2,5 / -2,5	0 / 0	-37 / -37	-30 / -30
	$f_{h\beta,1}$ / $f_{h\beta,2}$	0 / 0	-2,5 / 2,5	0 / 0	-37 / 35	-15 / 15
	$c_{\alpha,1}$ / $c_{\alpha,2}$	0 / 0	6 / 6	0 / 0	2 / 2	4 / 4
	$c_{\beta,1}$ / $c_{\beta,2}$	0 / 0	7 / 7	0 / 0	5 / 5	6 / 6

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Figure 8: Final Summary Diagram

The results for the gear design on the left side of the diagram do not include an optimized micro geometry and are in a quality class DIN 9. Stresses and Transmission error are on a very high level and spread in a wide range. Especially width angle deviations result in high stresses and a big transmission error. If the micro geometry is optimized by correcting the profile angle by $f_{h\alpha} = -2,5$ µm on both gears, the width angles by $f_{h\beta,pinion} = -2,5$ µm and $f_{h\beta,gear} = 2,5$ µm and adding crownings $c_\alpha = 6$ µm and $c_\beta = 7$ µm on both gears the values for surface stresses and transmission errors drop and the value range shrinks.
If the optimized gear design of quality DIN 9 is compared to an unmodified gear design of quality DIN 7 it can be observed that the values for stresses have nearly the same level, but the value range of the optimized gear in quality DIN 9 is smaller. For this gear design the mean values of the transmission error are slightly better, too. But they spread in a wider

range so that the process stability in this case is better for the unmodified gear design in quality DIN 7.
The results for the profile angle deviations $f_{h\alpha}$ at the gear design with optimised flanks and quality DIN 7 were already presented in this report. On the second position from the right side of the diagram in Figure 8 the results are summarised and supplemented with the results for the profile angle deviations $f_{h\beta}$. Compared to the previous described results, this gear design shows the best values for stresses and transmission error. Furthermore the values spread in a very narrow range.
On the far right side of the diagram is a gear design with optimized micro geometry and quality DIN 7 calculated for case hardened wrought steel. Compared to the results of the powder metal steel the stresses are a little bit higher. On the other hand the values for the transmission error are lower. For both, surface stresses and transmission error, the value range of the powder metal gear are narrower.

7 Summary and Outlook

The presented approach makes the micro geometry design of powder metal gears possible. Special material properties of powder metal, like density dependent values for Young's Modulus and Poisson's Ratio, can be considered as well as the surface densification at the flanks.
The ability to reduce surface stresses and transmission error could be shown by applying this method on the fourth gear of an automotive gear box. Both values could be reduced inside the whole range of different tolerance classes.
Furthermore the possibilities of PM as gear material were presented by comparing the achievable values for surface stress and transmission error to the equivalent values for a gear made of wrought steel.
The presented method can be implemented into the design process for powder metal gears. After defining the macro geometry of the gear the method can be used to define the micro geometry of the gear to improve load bearing capacity and noise vibration harshness to reduce the number of cost intensive tests with prototypes. Furthermore it can be used to improve the running behaviour of existing PM gear designs.

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